Section 10.7

Ratio Test

We are still want of more tests for convergence to handle those series for which are quick tests fail.

Theorem 10.7.1. Ratio Test
Let $\sum a_k$ be an infinite series, and let $r = \lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right|$.

1. If $r < 1$, the series converges absolutely.
2. If $r > 1$ (including $r \to \infty$), the series diverges.
3. If $r = 1$, the test is inconclusive.

Example 10.7.2. Utilize the Ratio Test.

1. $\displaystyle\sum_{k=1}^{\infty} \frac{10^k}{k!}$

2. $\displaystyle\sum_{k=1}^{\infty} (-1)^{k+1} e^{-k}(k^2+4)$

Root Test

Theorem 10.7.3. Root Test
Let $\sum a_k$ be an infinite series, and let $\rho = \lim_{k\to\infty} \sqrt[k]{|a_k|}$.

1. If $\rho < 1$, the series converges absolutely.
2. If $\rho > 1$ (including $r \to \infty$), the series diverges.
3. If $\rho = 1$, the test is inconclusive.

Example 10.7.4. Utilize the Root Test.

1. $\displaystyle\sum_{k=1}^{\infty} \left(\frac{k^2}{2k^2+1}\right)^k$

2. $\displaystyle\sum_{k=1}^{\infty} \left(1+\frac{3}{k}\right)^{k^2}$

Example 10.7.5. Test the following series for convergence.

1. $\displaystyle\sum_{k=1}^{\infty} \frac{k!}{(2k+1)!}$

2. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(1+p)^k}, p > 0.$

3. $\displaystyle\sum_{k=1}^{\infty} \frac{k}{e^k}$

4. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^{2k}}{k!k!}$

5. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{\arctan k}$

6. $\displaystyle\sum_{k=2}^{\infty} \frac{\ln k}{k^p}$